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Analysis of inventory system with three stages of deterioration

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Abstract - In this article, we consider a continuous review perishable inventory system with instantaneous replenishment policy. The status of perishable item in inventory is assumed to be in any one of the three stages good, average and damaged. The demand process is assumed to be Poisson, replenishment is instantaneous and the deterioration process is prescribed by certain transition probability matrix. Various stationary system performance measures are obtained. The total system maintenance cost rate is calculated and an optimal value of the S is obtained. The results are illustrated numerically.

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ANALYSIS OF INVENTORY SYSTEM WITH THREE STAGES OF DETERIORATION

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Analysis of inventory system with three stages of deterioration

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Abstract - In this article, we consider a continuous review perishable inventory system with instantaneous replenishment policy. The status of perishable item in inventory is assumed to be in any one of the three stages good, average and damaged. The demand process is assumed to be Poisson, replenishment is instantaneous and the deterioration process is prescribed by certain transition probability matrix. Various stationary system performance measures are obtained. The total system maintenance cost rate is calculated and an optimal value of the S is obtained.. The results are illustrated numerically.

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I. Introduction

The classical inventory theory did not take in to accounts items that have finite lifetime (deteriorating items). However, items stocked in real life situations are subject to perishability due to excessive storage time or because of technology/style of change (obsolescence) occur. Examples for perishable items include certain foods, chemical, medicines, seasonal products and so on. Analysis of inventory systems stocking perishable items has been the theme of many researchers in the last three decades. The often quoted review articles of Nahmias (1982) and Rafat (1991) provide excellent summaries of many of the perishable inventory models. A recent comprehensive review paper focusing on the management of items with finite shelf life is published by Karesman et al. (2009). According to their classification three different kinds of perishable inventory problems have been studied earlier. Continuous review models ; (i) without fixed ordering cost, zero lead time, (ii) without fixed ordering cost, positive lead time, (iii) with fixed ordering cost, zero lead time.

Category (i) problem was studied by Graves (1982), who assumed that items are continuously produced, perish after a deterministic time and that demand follows a compound Poisson process.

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Category (ii) problem was studied first by Pal (1989) who investigated the system with (S-1, S) policy. Nahimias et al. (2004), analyse the same type of problems with emphasise on the performance measures rather than cost optimization.

Category (iii) problem originated by Weiss (1980) is most relevant for our paper. Lian et al. (2005) considered discrete demand for items and time to perish is either fixed or that follows a phase type distribution.

Our model is almost close to this model in terms of deterioration process. We consider an inventory system in which each item has an exponential life time and deterioration process passes through three different stages (good, average and damaged). We indicate the states with numerical indices 1, 2 and 3 respectively. It is also assumed that damaged (completely unusable) items were removed from stock at review time (demand epoch). Demand process is assumed to be Poisson and the replenishment is instantaneous (zero lead time). Fixed ordering cost K and holding cost h are assumed.

II. Problem Formulation

Consider an inventory system which stocks perishable items whose life time is exponentially distributed having three states (sojourn time of each state is exponentially distributed). Demand process follows a Poisson process with unit demand at a time, Identify the three states as good, average and damaged with numerical indicators 1, 2 and 3 respectively. The damaged items have to be removed immediately after the current demand is satisfied at time points (review epochs). The following assumptions are made:

Demand process is assumed to be Poisson with parameter $\lambda > 0$.

The replenishment is assumed to be instantaneous with (0, S) policy, where S is the maximum inventory level. Transition from one state of the inventory item to another in the process of deterioration during demand processing time is a random- phenomena with transition probabilities are given by the matrix $P = (p_{ij}), i, j = 1,2,3.$

III. Model Description and Analysis

Let I (t) and S (t) denote the number of items in stock and the status of the perished items in inventory respectively at time ‘t’. Then {(I(t), S(t)); t >0 } is a two dimensional stochastic process with state space .

$$E = \begin{cases} (i, j) : i = S, S-1, S-2, \ldots\ldots, 1 \\ j = 1, 2, 3. \end{cases}$$

The embedded Markov chain $\{(I_n, S_n); n \geq 0\}$, where I_n denote the inventory level when the nth demand occurs and S_n, the status (stages) of the perishable item at that time.

The one step transition between stages of perishing is given by the tpm

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ 0 & P_{22} & P_{23} \\ 0 & 0 & 1 \end{bmatrix},$$

$$\textit{where } P_{11} = 1-(P_{12}+P_{13});\ P_{22} = 1-P_{23};\ P_{33} = 1.$$

Thus the two dimensional Markov process {(I(t), S(t)); t >0 } has transitions from one state to another as business cycle advances. From the assumptions we made on the input and output processes (replenishment and demand), it can be shown that the transition probabilities $\left(p_{(i,k)}^{(j,l)}(t)\right)$, of the Markov process has the derivative at time t=0. The intensity of transitions from state (i, k) to (j, l) is defined as $q_{(i,k)}^{(j,l)} = \frac{d}{dt} p_{(i,k)}^{(j,l)}(t) / t = 0.$

Now the infinitesimal generator, $Q = \left(q_{(i,k)}^{(j,l)}\right)$ of the Markov process be defined with intensity of transition defined as follows: System transition takes place from state:

- (i, k) to (i-1, k) with rate $\lambda > 0$ for k=1,2,3.
- (i, k) to (i-1, k+1) with rate $\lambda p_{i(k+1)}$ for k=1,2,3.
- (i, 3) to (S-1, 1) with rate $\lambda > 0$.

Infinitesimal generator (rate matrix) Q can be conveniently expressed as a block partitioned matrix $Q = (Q_{ij})$ where, i, j=1, 2, 3, …S.

$$Q_{ij} = \begin{cases} B & j = i = S \\ A & j = i, i = S-1, S-2, ..1 \\ \Lambda_P & j = i-1, i = S, S-1, ...2 \\ \Lambda_0 & j = S, i = S-1, ...2 \\ \Lambda_1 & i = 1, j = S. \\ 0, \textit{otherwise}. \end{cases}$$

More explicitly,

$$Q = \begin{bmatrix} B & \wedge_P & 0 & 0 & \ldots & 0 \\ \wedge_0 & A & \wedge_P & 0 & \ldots & 0 \\ \wedge_0 & 0 & A & \wedge_P \ldots & & 0 \\ \wedge_0 & 0 & 0 & A & \ldots. & \Lambda_p \\ \vdots & \vdots & \vdots & \vdots \ldots & & \vdots \\ \wedge_1 & 0 & 0 & 0 \cdots & & A \end{bmatrix}$$

Where,

$$B = \begin{bmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ \lambda & 0 & -\lambda \end{bmatrix}$$

$$\wedge_P = \begin{bmatrix} \lambda P_{11} & \lambda P_{12} & \lambda P_{13} \\ 0 & \lambda P_{22} & \lambda P_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{bmatrix} = -\lambda I$$

$$\wedge_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \lambda & 0 & 0 \end{bmatrix}.$$

$$\wedge_1 = \begin{bmatrix} \lambda & 0 & 0 \\ \lambda & 0 & 0 \\ \lambda & 0 & 0 \end{bmatrix}$$

Since the state space of the Markov chain is finite, the states of the system are recurrent non-null and aperiodic. So by limiting probability arguments the Markov chain embedded in the process is ergodic. Let the steady state probability distribution of the states of the system (π_j), exists and can be obtained by solving the matrix equation

$$\pi Q = 0.$$

Let $\pi = (\pi_S, \pi_{S-1}, \ldots, \pi_1)$, where each $\pi_j = \left(\pi_j^{(1)}, \pi_j^{(2)}, \pi_j^{(3)}\right)$, with reference to the perished state of the system. Here the inventory means the on hand inventory, (not the position inventory). Now we get a system of matrix equations

$$\pi_S B + \sum_{i=1}^{S-2} \pi_{S-i} \wedge_0 + \pi_1 \Lambda_1 = 0 \quad and$$

$$\pi_j \Lambda_p + \pi_{j-1} A = 0, j = S, S-1, \ldots, 2.$$

Assuming the initial probability vector $\pi_S = \left(\pi_S^{(1)}, \pi_S^{(2)}, \pi_S^{(3)}\right)$, we are able to get the solution for the above system of equations using recurrence method together with normalizing equation

$$\sum_{j=1}^{3} \left(\sum_{i=1}^{S} \pi_i^{(j)} \right) = 1.$$

The solution in terms of π_S is given by

$$\pi_k = (-1)^{S+k} \pi_S (\Lambda_P)^{S-k} A^{-S+k}, \; 1 \le k \le S-1.$$

And $\pi_S = 1' \left(I + \sum_{j=1}^{S-1} M_j \right)^{-1}$, where $1' = (1,1,1)$ and

$$M_K = (-1)^{S+k} (\Lambda_p)^{S-k} A^{-S+k}.$$

IV. System Performance Measures

The system performance measures of the perishable inventory system we considered can be obtained using the steady state probability vector $\pi_i^{(j)}, i = 1,2,3,\ldots S \text{ and } j = 1,2,3.$

a) Mean inventory level

Let $\overline{L}$ denote the mean inventory level of the system in steady state. Then

$$\overline{L} = \sum_{i=1}^{S} \left(\sum_{j=1}^{3} i \pi_i^{(j)} \right).$$

b) Mean Reordering rate

The inventory maintained in the system is of perishable nature with three different states j = 1, 2, 3, and the last state j =3 represent the 'damaged' state of the items in inventory. All items with this condition can be removed from the system immediately after the supply of the demanded item to the customer. Simultaneously the instantaneous replenishment takes place for S items with zero lead time ((0, S) policy assumed). By the above arguments we can establish that the reorder rate 'β' for the system is given by

$$\beta = \left(\sum_{j=1}^{3} \pi_1^{(j)} + \sum_{i=2}^{S} \pi_i^{(3)} \right) \lambda.$$

V. Cost Optimization

The total cost incurred for the proposed system can be obtained with the assumption of proper cost structure as follows.

1. The reorder cost be 'K' per order per unit time.
2. The holding cost is 'h' per item per unit time.

Thus the total cost per unit time [total cost rate] is given by

$$TCU(S) = h\overline{L} + K\beta$$

$$= h \sum_{i=1}^{3} \left(\sum_{j=1}^{3} i \pi_i^j \right) + K \left(\sum_{j=1}^{3} \pi_1^{(j)} + \sum_{i=2}^{S} \pi_i^{(3)} \right) \lambda.$$

VI. Numerical Examples

The convexity of the total cost rate function TCU(S) cannot be proved analytically, due to its complex form. Hence a detailed computational study of the cost function is carried out and try to get the optimal solution S*(optimum ordering quantity) by implementing proper searching algorithms.The criterion used here is the minimization of total expected cost rate. Consider the numerical example with following parameters and transition probability matrix for the deterioration process:

$$P = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0 & 0.7 & 0.3 \\ 0 & 0 & 1 \end{bmatrix}.$$

The system parameters S, λ, and the cost parameters K and h are varied and the corresponding expected total cost rates are obtained using the system performance measures we derived in the previous section IV.

All cost rates together with its optimal values are given in tables-1, 2 and 3. Table-1 shows the expected total costs for fixed parameters K= 100 and h=3, S varied from 4 to 10 and λ varied from 2 to 5. The local convex nature of the total expected cost rate function yields the following optimal pair of system parameters

(λ, S*): {(2, 8), (3, 6), (4, 5), (5, 5)}.

Table-2 shows the expected total costs for fixed parameters λ =2, h =3, S varied from 4 to 10 and K varied from 25 to 100 with step 25. The local convex nature of the expected cost rate function yields the following optimal pair of system parameters:

(K, S*): {(25, 5), (50, 6), (75, 7), (100, 8)}.

Table -1 (K = 100, and h = 3)

S \ λ	2	3	4	5
	Total expected cost rate			
4	283.5396106	365.7873845	455.3403856	549.2222358
5	267.0650956	351.9852187	**446.6706657**	**544.6869059**
6	258.4846614	**349.7303686**	449.1308320	549.7077697
7	254.4437136	352.9306701	455.6550818	557.5243486
8	**253.2166870**	358.8100108	463.6914970	566.1660979
9	253.8151260	366.0487049	472.3026071	575.0547297
10	255.6312430	374.0032820	481.1386357	584.0188567

Table -2 (λ= 2, and h = 3)

S \ K	100	75	50	25
	Total expected cost rate			
4	283.5396106	218.2797080	153.0198053	87.75990265
5	267.0650956	207.0488217	147.0325478	**87.01627391**
6	258.4846614	201.7384961	**144.9923307**	88.24616535
7	254.4437136	**199.8327852**	145.2218568	90.61092840
8	**253.2166870**	200.0375153	146.8583435	93.67917178
9	253.8151260	201.6113445	149.4075630	97.20378150
10	255.6312430	204.0984323	152.5656215	101.0328108

Table-3 (K =100, and λ = 2)

S \ h	3	4	5	6
4	283.5396106	291.0396106	298.5396106	306.0396106
5	267.0650956	276.0650956	285.0650956	294.0650956
6	258.4846614	268.9846614	279.4846614	**289.9846614**
7	254.4437136	**266.4437136**	**278.4437136**	290.4437136
8	**253.2166870**	266.7166870	280.2166871	293.7166871
9	253.8151260	268.8151260	283.8151260	298.8151260
10	255.6312430	272.1312430	288.6312431	305.1312431

Table-3 shows the expected total costs for fixed parameters, K= 100 and λ = 2, S varied from 4 to 10 and h varied from 3 to 6. The local convex nature of the total expected cost rate function yields the following optimal pair of system parameters:

(h, S*): {(3, 8), (4, 7), (5, 7), (6, 6)}.

The above sensitivity analysis shows that the expected total cost rate is sensitive to both the system parameters S, λ and the cost parameters K and h.

VII. Conclusion

Here we formulated and analyzed a perishable inventory system with (0, S) policy. The different stages of deterioration are considered (good, average, damaged). This model dealing with perishable inventory control system is tractable because of the assumed instantaneous replenishment ((0, S) policy). But in reality there exist a positive lead time. So further work in direction is possible by generalizing the policy to (s, S) type with ordering quantities Q= S - s .This batch ordering policy may increase the complexity of the problem and hence the tractability of the system becomes a question.

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